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MRI

POLYTECHNIC INSTITUTE OF BROOKLYN

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## ABSTRACT

The technique for interchanging generator and detector in the impedance measurement of microwave one-ports is a useful, known procedure often applied when low powers are indicated. The necessary and sufficient conditions for the validity of such measurements are critically examined and direct extensions of this technique to similar measurements of reciprocal two-ports are given.

A completely separate analysis is necessary when such an interchange is made in the case of an interference bridge to be used for the determination of the scattering parameters of general (active or passive and reciprocal or nonreciprocal) two-ports. This analysis is presented in detail. It results in a new low-power-level version of a method of measuring general two-ports given in an earlier paper.[5] The measurement technique and the subsequent data analysis of the two versions are found to be identical, except that the two scattering parameters $S_{12}$ and $S_{21}$ appear in interchanged positions.

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## I. Introduction

In the (one-port) measurement of the impedance characteristics of such devices as diodes for which powers may have to be held below certain maximum levels, it has become common practice to interchange generator and detector in order to increase the power to the detector substantially above the level it would be in a normal impedance measuring set-up.[1] This approach has been suggestive with regard to bridge techniques for measuring two-port parameters under similar limitations of maximum allowable power levels. In particular such an approach would be useful in certain measurements which seek to avoid regions of saturation or of nonlinearity.

In what follows, the standard one-port technique is first briefly reviewed and then more critically examined. Its extension to two-port impedance measurements is also indicated. The technique to be used in a reciprocal bridge for the measurement of reciprocal two-ports is pointed to and finally the method associated with source-detector interchange in the measurement of general(potentially non-reciprocal and active) linear two-ports by means of a practical interference bridge, i. e.,one which also contains isolators, is derived. For this last method a step by step summary of the measurement is given in Appendix III.

## II. Interchange in Impedance Measurements

### A. One-Port Measurements

The common technique for measuring the impedance of an unknown load is depicted in Fig. 1. The notion of a matched detector here implies that while energy

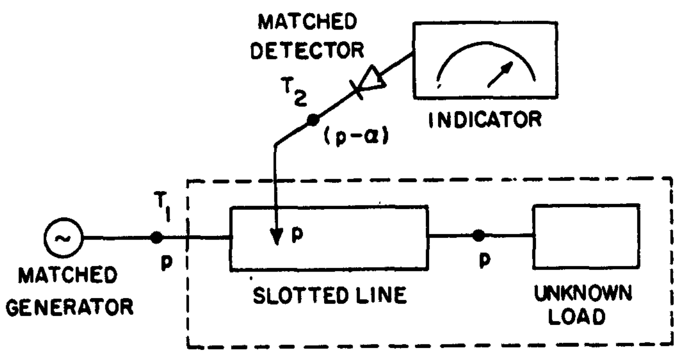


Fig. 1 Common impedance measurement

may be reflected by the probe, or even at a point beyond it, no microwave energy may be reflected by the detector itself. Both the detector and the generator may be taken to include any necessary tuners or isolators. The structure contained within dashed lines in Fig. 1, including the unknown load, can be regarded as a reciprocal two-port between terminal planes $T_1$ and $T_2$. Its parameters, of course, depend on the load and the location of the probe. In view of the reciprocity of the two-port ($S_{12} = S_{21}$) and the match of the generator and the detector, the reading on the indicator remains completely unaltered when these two components are interchanged as shown in Fig. 2. Since this is true for any such two-port, i.e., for any load or probe position, measurements may be carried out exactly as if the interchange had not been made.

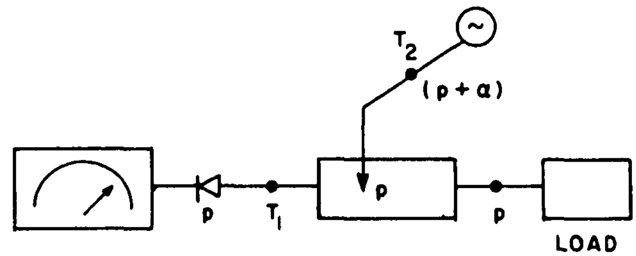


Fig. 2 Source-detector reversal in load impedance measurement

In Fig. 1 the generator power has been set at a level p (say in dbm), power at the probe and the load is also at p; however, in view of a db decoupling of the probe, power at the detector is (p - a). In contrast, in the set-up of Fig. 2, generator power is (p + a) so that the level at the probe and the load is again p as before, but power at the detector is now p, no longer (p - a).

The virtue of this method then lies in the fact that while the power level is maintained at the load within imposed bounds, the level at the detector has been increased by a db, where a presumably represents some minimum probe decoupling. Advantage can be taken of any additional available generator power by further decoupling the probe as far as this power will permit. Two distinct advantages have now been seen to be possible: Additional sensitivity (by a db), and greater accuracy as the result of the reduction of the usual errors which are associated with probe reflection to an extent which depends on available power.

As can be expected, a price must be paid for what has been gained. When the probe is moved in the course of the measurement, a flexible waveguide must be used in Fig. 2 instead of the audio cable of Fig. 1. The flexible waveguide may introduce unwanted leakage or additional small probe-position dependent reflections. When this becomes intolerable two microwave-wise sound but mechanically troublesome alternatives have been used: Either the generator (tube) can be connected rigidly to the probe carriage and moved with it, or the generator and probe can be held fixed while the slotted line, detector and load are moved as a unit with respect to the probe.

### B. Extension to Two-Port Measurements

A useful feature of this scheme is the fact that any measurement technique involving equipment which can somehow be viewed as a reciprocal microwave two-port connected to a matched source at one port and to a matched detector at the other, is subject to source-generator interchange without in the least affecting the method of measurement or the associated data analysis: No further thought is required and no new formulas need be derived. For example, the Weissfloch[2] method of measuring reciprocal two-ports* or the Deschamps[3] method of finding the scattering coefficients of reciprocal two-ports can be used, and perhaps to advantage, with source and detector interchanged.

### C. Necessary and Sufficient Conditions for Source-Detector Interchange

Although the restriction of matched generator and detector is doubtlessly sufficient, the necessary conditions should also be explored. As will be seen, these extend somewhat the usefulness of the technique. Consider a nonreciprocal two-port connected to a generator with internal impedance $Z_g$ and to a detector of impedance $Z_d$ as shown in Fig. 3.

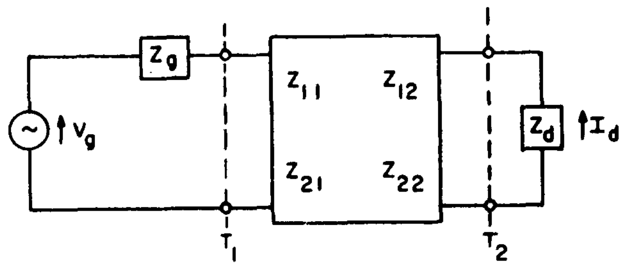


Fig. 3 Two-port with source and detector

Detector response is proportional to $(I_d/V_g)^2$ so that source and detector may be interchange provided

$$\left[\frac{I_d/V_g}{I'_d/V_g}\right]^2 = \text{constant}, \tag{1}$$

as the two-port parameters are varied in the course of a measurement. Here $I'_d$ is the detector current after the interchange has been made. Clearly from eq. (1)

---

* These, by the way, may be active if the first resistance in the representation is appropriately taken as either positive or negative. The distinction between reflection coefficients greater and less than unity must be made in the course of the measurement.

$$\frac{I_d/V_g}{I'_d/V_g} = \text{constant} \tag{2}$$

is also a necessary and sufficient condition. It is convenient to recognize that $Z_g$ and $Z_d$ are associated with the diagonal elements of the impedance matrix as shown Fig. 4, which indicates only the relevant voltages and currents. From Fig. 4 one has

$$\begin{aligned} V_g &= (Z_{11} + Z_g)I_g + Z_{12}\, I_d \\ V = 0 &= Z_{21}I_g + (Z_{22} + Z_d)I_d \end{aligned} \tag{3}$$

and when $I_g$ is eliminated from eq. (3) the result is

$$I_d/V_g = \frac{Z_{21}}{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{11}Z_d + Z_{22}Z_g)} \tag{4}$$

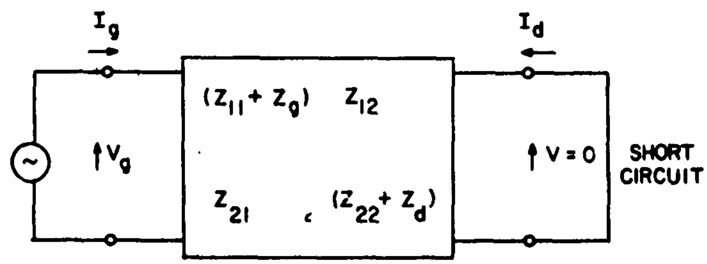


**Fig. 4** Two-port incorporating generator and detector impedances

One sees from Fig. 2 that the interchange can be simply represented by interchanging subscripts 1 and 2, so that, from eqs. (2) and (4), the desired condition is

$$\frac{Z_{21}}{Z_{12}} \cdot \frac{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{22}Z_d + Z_{11}Z_g)}{(Z_{12}Z_{21} - Z_{11}Z_{22} - Z_dZ_g) - (Z_{11}Z_d + Z_{22}Z_g)} = \text{constant}. \tag{5}$$

Since the two-port parameters have been assumed to be arbitrary, and since their variation in the course of a measurement has not been restricted, eq. (5) shows that the necessary and sufficient condition for source-detector interchange is

$$Z_d = Z_g \quad \text{provided} \quad Z_{21}/Z_{12} = \text{constant}. \tag{6}$$

This proviso holds in most practical measurement situations in view of the following theorem which is demonstrated in Appendix I:

> The product of the nonreciprocities of tandem two-ports equals the nonreciprocity of the overall two-port.

The nonreciprocity of a two-port $\mathfrak{Z}$ is here taken as $\sqrt{\mathfrak{Z}_{12}/\mathfrak{Z}_{21}}$. The nonreciprocity of a reciprocal two-port is, of course, unity. It is clear then that if the two-port Z is comprised of an arbitrary number of cascaded two-ports, $Z_{21}/Z_{12}$ is constant as long as the only two-ports in the chain which vary in the course of the measurement are reciprocal. An example of such a situation would be an isolator at $T_1$ in tandem with the slotted line in Figs. 1 and 2.

It can be shown quite independently that eq. (6) in scattering terms is

$$\Gamma'_d = \Gamma_g, \quad \text{provided } S_{21}/S_{12} = \text{constant,} \tag{7}$$

where $\Gamma_d$ and $\Gamma_g$ are the reflection coefficients of the detector and the generator, respectively. This, of course, also follows simply from the recognition that $\Gamma_d$ and $\Gamma_g$ are the input reflection coefficients corresponding to the detector and generator input impedances $Z_d$ and $Z_g$ and from the fact[4] that $S_{21}/S_{12} = Z_{21}/Z_{12}$ for two-ports.

Equations (5) to (7) are quite general and as such apply also to the interchange pictured in Figs. 1 and 2 involving a slotted line. When, in that case, a well tuned and decoupled probe is employed, the following approximations can usually be made:

$Z_{11}$ is constant as the probe is moved.
$Z_{22}$ is constant as the probe is moved.
$Z_{12}Z_{21}$ is sufficiently small to be neglected.

It is seen that under these conditions eq. (5) holds without any restrictions having been placed an $Z_g$ and $Z_d$ provided, again, the ratio $Z_{21}/Z_{12}$ is a constant as explained before.

## III. Interchange in Impedance Measurements

### A. Reciprocal Case

Microwave bridge measurements are used both in situations involving nonreciprocal as well as reciprocal two-ports as unknowns. When the latter is the case and when the bridge itself, as distinct from the detector and the generator arms, does not contain nonreciprocal components, the results given for the general case in eqs. (6) and (7) of course hold as soon as a two-port connecting source and detector can be identified. Fig. 5, which pictures a simplified outline of an interference bridge, identifies this two-port as being located between planes $T_1$ and $T_2$.

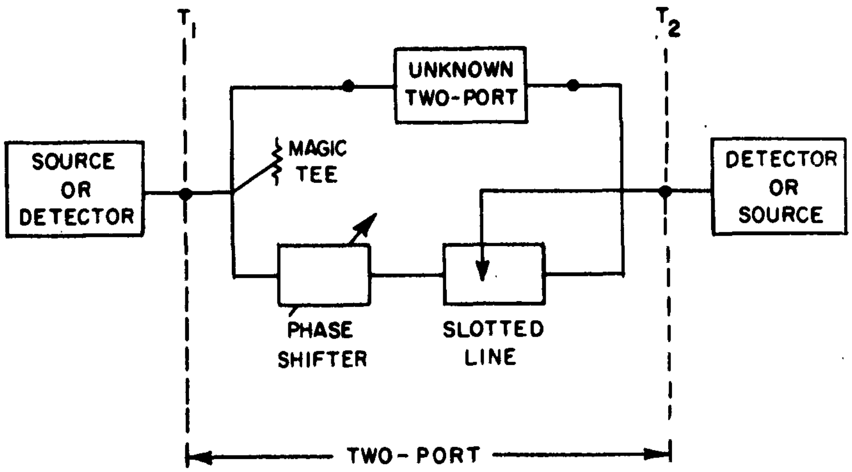


Fig. 5 Source-detector reversal with reciprocal bridge

## B. Nonreciprocal Case

When a microwave bridge has internal nonreciprocal components, such as isolators, or when it is used for the measurement of nonreciprocal structures, the preceding reciprocity argument can no longer be used when source and detector are interchanged. One can, however, seek the answer directly to the question of what actually does happen when the interchange is made in such a case. This is done below for an interference bridge. The measurement which evolves will be seen to have the same advantages with respect to power levels as the corresponding impedance measurement discussed above.

The two forms of the bridge which are sketched in Figs. 6 and 7 will be referred to as the "forward" bridge and the "reversed" bridge, respectively. Note that generator (including the associated isolator) and detector have been interchanged and that the two isolators adjacent to the slotted lines have been reversed. In each set-up only one probe at a time is in use. Since the forward bridge has already been amply treated[5], it will not be discussed here again.

### 1. Parameters of the Data Loci

Consider the reversed bridge: Two waves $a_1'$ and $a_2'$ are set up in the left slotted line by the probe and are traveling in the directions indicated. As a result three distinct waves reach, and add at, the detector. The voltage wave traveling from the probe directly to the left is

$$V_1 = a_1' C_1 e^{j\kappa z_1} \quad , \tag{8}$$

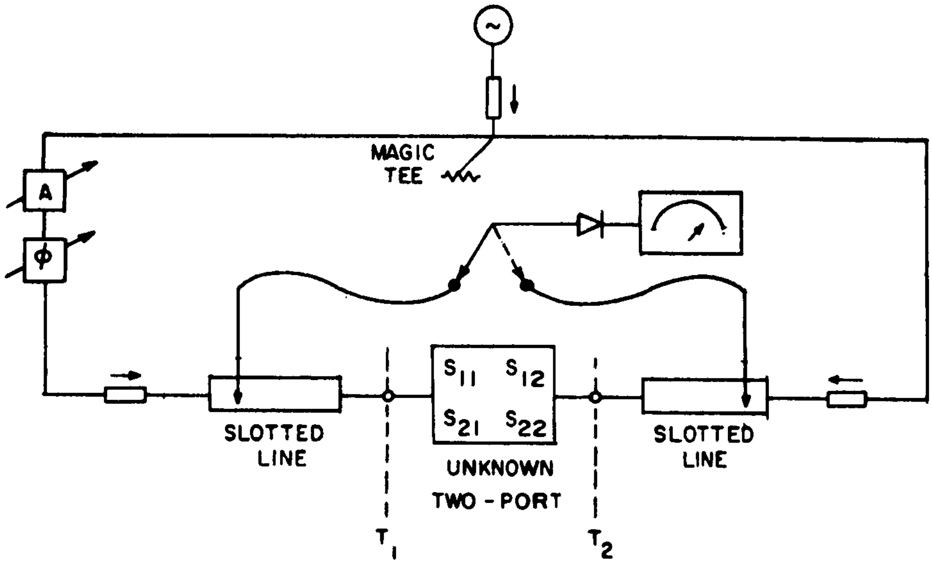


**Fig. 6 Forward bridge**

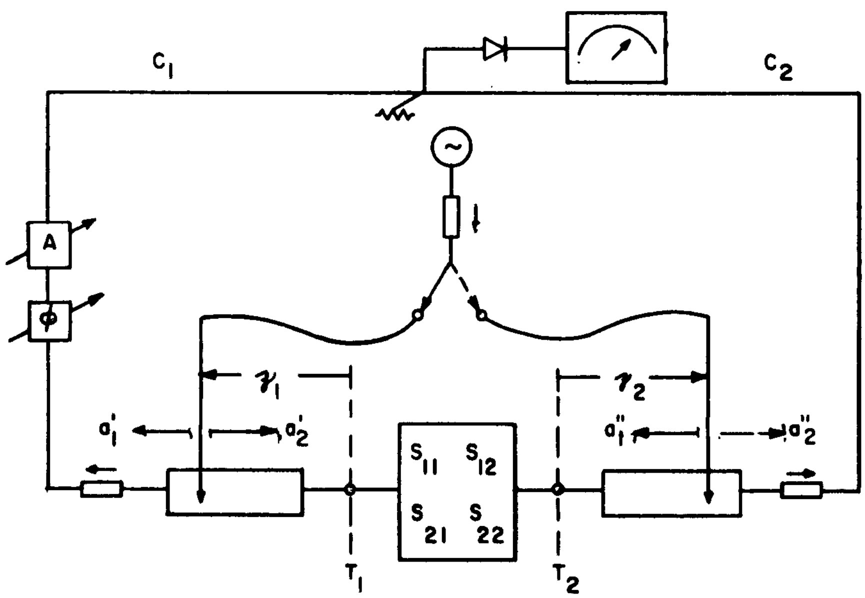


**Fig. 7 Reversed bridge**

where $C_1$ is the overall transmission coefficient through the left bridge arm to the detector arm and includes the effect of the variable phase shifter $\phi$ and the variable attenuator A. The wave from the probe to the right which is transmitted through the two-port is

$$V_2 = a_2' C_2 e^{-j\kappa z_1} S_{21} \,, \tag{9}$$

where $C_2$ is analogous to $C_1$, but a constant. The wave from the probe to the right which is reflected by the two-port is

$$V_3 = a_2' C_1 \, e^{-j\kappa z_1} S_{11} . \tag{10}$$

The voltage at the input of the detector, $V_d$, then is the sum of $V_1$, $V_2$ and $V_3$

$$V = a_2' \, e^{-j\kappa z_1} (S_{11} C_1 + S_{21} C_2) + a_1' \, e^{j\kappa z_1} C_1 \tag{11}$$

which can also be written

$$V_d = a_1' C_1 \left[ e^{-j\kappa z_1} \frac{a_2'}{a_1'} \left(S_{11} + \frac{C_2}{C_1} S_{21}\right) + e^{j\kappa z_1} \right] . \tag{12}$$

Since only a voltage proportional to the magnitude of $V_d$ is of interest, one need only consider

$$V_d' = e^{-j\kappa z_1} \frac{a_2'}{a_1'} \left(S_{11} + \frac{C_2}{C_1} S_{21}\right) + e^{j\kappa z_1} . \tag{13}$$

In view of the definition of $z_1$ (increasing from right to left in Fig. 7), the $\exp(j\kappa z_1)$ and $\exp(-j\kappa z_1)$ terms are respectively incident (from the left) on plane $T_1$ and reflected at plane $T_1$. The coefficient of $\exp(-j\kappa z_1)$ is consequently identified as the reflection coefficient $\Gamma_1$ at $T_1$ (defined looking to the right). As phase shifter $\phi$ is varied, the phase of $C_1$ varies with it and $\Gamma_1$ is seen to be a circular locus. If the probe is perfectly symmetrical, i.e., straight, as one attempts to make it, then $a_1' = a_2'$ and

$$\Gamma_1 = S_{11} + \frac{C_2}{C_1} S_{21} . \tag{14}$$

The conditions under which $a_1 = a_2$ are discussed in Appendix II. As in the case of the forward bridge the $\Gamma_1$ locus has its center at $S_{11}$, but now has the radius $|S_{21}|$, provided the bridge has been balanced by adjustment of variable attenuator A for $|C_1| = |C_2|$.

Analogously for probe 2, with $z_2$ defined as increasing to the right:

$$V_d'' = e^{-j\kappa z_2} \frac{a_1''}{a_2''} \left(S_{22} + \frac{C_1}{C_2} S_{12}\right) + e^{j\kappa z_2} \tag{15}$$

and

$$\Gamma_2 = S_{22} + \frac{C_1}{C_2} S_{12} \tag{16}$$

where $\Gamma_2$ is the reflection coefficient at plane $T_2$ defined looking to the left. The locus of $\Gamma_2$ is again a circle, but with center at $S_{22}$ and, when the bridge has been balanced, with radius $|S_{12}|$.

In terms of traveling waves, the balance condition of a reversed bridge is the following: If $a_1$ equals $a_2$, and $a_1$ is defined at $T_1$ and traveling to the left while $a_2$ is defined at $T_2$ and traveling to the right, then, if detector voltage is zero when these two waves exist at the same time, the bridge is balanced, i. e., $|C_1| = |C_2|$. Some reflection shows that the various ways of balancing a forward bridge[5] apply completely in this case also.

## 2. Reference Plane and Phase Measurements

When the scale with which $z_1$ is measured is the D scale so that

$$z_1 = D - D_R \tag{17}$$

the reference value $D_R$, which is defined with respect to $T_1$, is found in the same fashion as if the bridge were a forward bridge. This follows readily from the fact that such measurements no longer are actual bridge measurements, but are impedance measurements of the sort discussed earlier. Likewise, when the scale with which $z_2$ is measured is the S scale so that

$$z_2 = S - S_R , \tag{18}$$

the reference value $S_R$, which is defined with respect to $T_2$, is again found as in a forward bridge.

Equations (13) and (15) are both of the form

$$V_d = e^{-j\kappa z}\Gamma + e^{j\kappa z}, \qquad \Gamma \equiv |\Gamma| e^{j\theta}. \tag{19}$$

How can such a value of $\Gamma$ be determined with a reversed bridge? The probe can be moved to two distinct locations such that $|V_d|$ is a maximum ($V_M$) or a minimum ($V_m$). Let the probe setting associated with $V_m$ be $z_m$. Then

$$V_M = |\Gamma| + 1 \tag{20}$$

$$V_m = \Big|\, |\Gamma| - 1 \Big| = \Big| |\Gamma| e^{j(\theta - \kappa z_m)} + e^{j\kappa z_m} \Big| \tag{21}$$

so that from eq. (21),

$$\theta = 2\kappa z_m \pm \pi \quad . \tag{22}$$

But since $z_1$ and $z_2$ are given by eqs. (17) and (18),

$$\theta_1 = 2\kappa(D - D_R) \pm \pi, \tag{23}$$

$$\theta_2 = 2\kappa(S - S_R) \pm \pi, \tag{24}$$

where D and S must now be interpreted as the probe positions when $V'_d$ and $V''_d$ are minimum. From eqs. (20) and (21)

$$|\Gamma| = \frac{r - 1}{r + 1}, \quad r \equiv \pm V_M/V_m. \tag{25}$$

Since $|\Gamma|$ may either be greater or less than unity, the standing wave ratio, r, is taken to be positive for $|\Gamma| < 1$ and negative for $|\Gamma| > 1$. Means of deciding whether $|\Gamma|$ is greater or less than unity, which apply here also, are given elsewhere.[5]

When the unknown two-port of Fig. 7 is replaced by a waveguide of length $\ell_o$ so that no reflections occur at $T_1$ or at $T_2$ and when the phase shifter is set at some arbitrary but fixed setting $\phi_o$, then each probe in turn can be moved to such a position that there will be minimum detector output (zero when $a_1/a_2 = 1$ and $|C_1/C_2| = 1$). Let these probe settings be defined as $D_o$ and $S_o$. In view of the fact that the scattering matrix of line $\ell_o$ is

$$\begin{pmatrix} 0 & e^{-j\kappa\ell_o} \\ e^{-j\kappa\ell_o} & 0 \end{pmatrix} \tag{26}$$

and with the assumption that $a_1 = a_2$, detector voltages $V_d$ in eqs. (13) and (15) will be at a minimum when

$$e^{-j\kappa(D_o - D_R)} e^{j(\phi_2 - \phi_{1o} - \kappa\ell_o)} = - e^{j\kappa(D_o - D_R)} \tag{27}$$

and when

$$e^{-j\kappa(S_o - S_R)} e^{j(\phi_{1o} - \phi_2 - \kappa\ell_o)} = - e^{j\kappa(S_o - S_R)}, \tag{28}$$

where $C_2$ has the phase $\phi_2$ and $C_1$ the phase $\phi_1$. When, as is the case now, the phase shifter is set to $\phi_o$, the angle $\phi_1$ takes on the value $\phi_{1o}$. From eqs. (27) and (28) one readily obtains

$$\phi_2 - \phi_{1o} = 2\kappa\,(D_o - D_R) + \kappa\ell_o \pm \pi \tag{29}$$

$$\phi_2 - \phi_{1o} = - \left[ 2\kappa(S_o - S_R) + \kappa\ell_o \pm \pi \right] . \tag{30}$$

On equating eqs. (22) and (23) one finds that

$$D_o + S_o + \ell_o - D_R - S_R = \frac{n\lambda_g}{2}. \tag{31}$$

Equation (31) can serve as a consistency check on the measurements made in finding $D_o$, $S_o$, $\ell_o$, $D_R$ and $S_R$.

It is convenient to plot the measured data in the $\overline{\Gamma}_1$ and $\overline{\Gamma}_2$ planes where

$$\overline{\Gamma}_1 \equiv |\Gamma_1| \; e^{j2\kappa D} = |\Gamma_1| \, e^{j\overline{\theta}_1} = \Gamma_1 \, e^{j(2\kappa D_R \pm \pi)} \tag{32}$$

$$\overline{\Gamma}_2 \equiv |\Gamma_2| \; e^{j2\kappa S} = |\Gamma_2| \, e^{j\overline{\theta}_2} = \Gamma_2 \, e^{j(2\kappa S_R \pm \pi)} \tag{33}$$

It follows from these equations and eqs. (14) and (16) that

$$\begin{aligned} \overline{\Gamma}_1 &= |S_{11}| e^{j(\phi_{11} + 2\kappa D_R \pm \pi)} + \left| \frac{C_2}{C_1} S_{21} \right| e^{j(\phi_{21} + 2\kappa D_R \pm \pi + \phi_2 - \phi_1)} \\ &\equiv |S_{11}| e^{j\overline{\phi}_{11}} + \left| \frac{C_2}{C_1} S_{21} \right| e^{j\psi_1} \end{aligned} \tag{34}$$

$$\begin{aligned} \overline{\Gamma}_2 &= |S_{22}| \, e^{j(\phi_{22} + 2\kappa S_R \pm \pi)} + \left| \frac{C_1}{C_2} S_{12} \right| e^{j(\phi_{12} + 2\kappa S_R \pm \pi - \phi_2 + \phi_1)} \\ &\equiv |S_{22}| \, e^{j\overline{\phi}_{22}} + \left| \frac{C_1}{C_2} S_{12} \right| e^{j\psi_2} \end{aligned} \tag{35}$$

and it is therefore easy to identify

$$\phi_{11} = \overline{\phi}_{11} - 2\kappa D_R \pm \pi \tag{36}$$

$$\phi_{22} = \overline{\phi}_{22} - 2\kappa S_R \pm \pi. \tag{37}$$

When, with the two-port to be measured located in the bridge, the phase shifter is set to the same value $\phi_o$ which was used earlier, $\phi_1$ again takes on the value $\phi_{1o}$, $\psi_1$ and $\psi_2$ become the specific angles $\psi_{1o}$ and $\psi_{2o}$, and $\overline{\Gamma}_1$ and $\overline{\Gamma}_2$ are the measured points $\overline{\Gamma}_{1o}$ and $\overline{\Gamma}_{2o}$. Under these conditions one identifies

$$\phi_{21} = \psi_{1o} - 2\kappa D_R \pm \pi - \phi_2 + \phi_{1o} \tag{38}$$

$$\phi_{12} = \psi_{2o} - 2\kappa S_R \pm \pi - \phi_2 - \phi_{1o}. \tag{39}$$

On using eqs. (29) and (30) in eqs. (38) and (39), respectively, one has

$$\phi_{21} = \psi_{1o} - 2\kappa D_o - \kappa \ell_o \tag{40}$$

$$\phi_{12} = \psi_{2o} - 2\kappa S_o - \kappa \ell_o. \tag{41}$$

Figure 8 shows the $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ loci explicitly in the sense of eqs. (34) and (35). The points $\bar{\Gamma}_{1o}$ and $\bar{\Gamma}_{2o}$ and the associated angles $\psi_{1o}$ and $\psi_{2o}$ are also shown. The radii

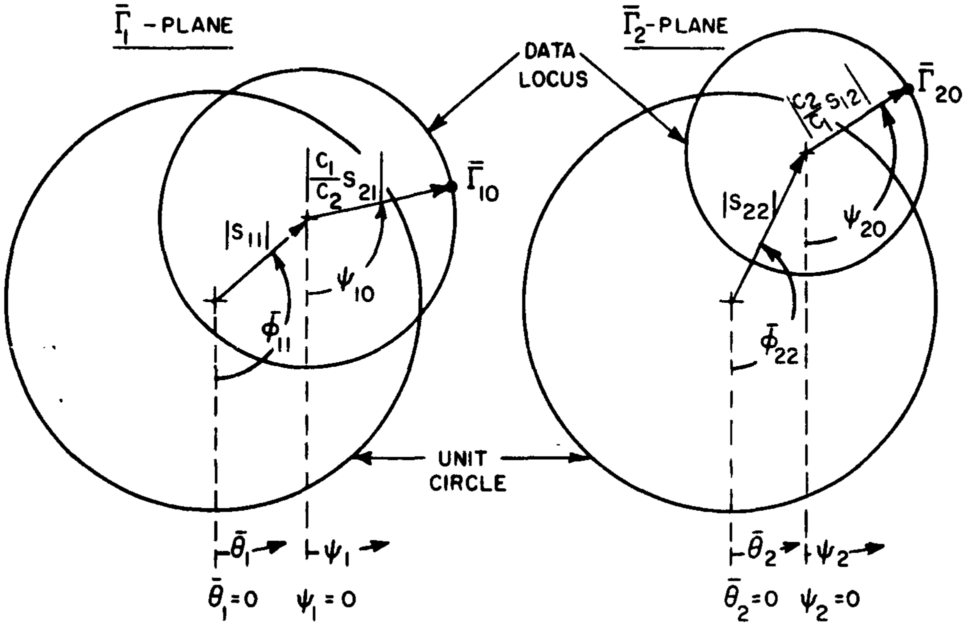


Fig. 8 Reflection coefficient loci obtained with reversed bridge

of course yield $|S_{21}|$ and $|S_{12}|$ directly when the bridge has been balanced ($|C_1| = |C_2|$).

The measurement procedure to be used with the reversed bridge is summarized in Appendix III.

### 3. Comparison of Measurements with Forward and Reversed Bridges

When one compares measurements made with a forward bridge with those made with a reversed bridge, one finds that the following simple generalization can be made: If a measurement and data analysis has been carried out with one bridge to determine the scattering matrix parameters of a two-port then, if precisely the same measurement and analytical steps are followed with the other bridge, the same matrix elements are obtained, but in transposed order, i. e., $\tilde{S}$ instead of $S$. In other words, $|S_{11}|$, $\phi_{11}$, $|S_{22}|$ and $\phi_{22}$ result from the same steps in both cases, but the steps which produce $|S_{12}|$ and $\phi_{12}$ in one case produce $|S_{21}|$ and $\phi_{21}$ in the other.

### 4. Reversed Bridge and Various Two-Port Representations

Since, when matrices Z or Y correspond to S then $\tilde{Z}$ or $\tilde{Y}$ correspond to S, one can make the following statement: Any measurements performed with a forward bridge of the type considered to result in Z or Y, will consequently result in $\tilde{Z}$ or $\tilde{Y}$ when performed with a reversed bridge. Other nonreciprocal representations such as the scattering transfer matrix, the voltage current transfer matrix (ABCD), or the nonreciprocal modified Wheeler network, if they can be obtained by the use of such bridges, are not, however, subject to similar matrix transposition. Instead it is recognized that the nonreciprocal character of such a representation is described by its nonreciprocity, K, and that the notion of the transposition of the S matrix of a two-port is completely described by the inversion of the nonreciprocity (see eq. (44)). This notion can therefore be used whenever matrix transposition is no longer in place. For example, in the case of the ABCD matrix (see eq. (42)), if A, B, C and D would be the parameters obtained by means of a forward bridge, the same measurement with a reversed bridge will yield $A/K^2$, $B/K^2$, $C/K^2$ and $D/K^2$. The converse, of course, is also true.

## Appendix I: Product of Nonreciprocities

It has been shown[6] in voltage-current transfer matrix terms that a nonreciprocal two-port can be decomposed into a reciprocal two-port and a special non-reciprocal two-port (ratio repeater) as follows

$$M \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_o & B_o \\ C_o & D_o \end{pmatrix} \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} \equiv M_o \mathcal{J}, \quad (42)$$

where ABCD is the nonreciprocal two-port and $A_oB_oC_oD_o$ the reciprocal two-port, i. e., where $\sqrt{A_oD_o - B_oC_o} = 1$. The ratio repeater is seen to be represented by a scalar matrix $\mathcal{J}$ with the associated scalar $K \equiv \sqrt{AD - BC}$. Clearly, when several two ports (indicated by different superscripts) are in tandem, then

$$M^1M^2 \ldots M^n = M_o^1\mathcal{J}^1 \cdot M_o^2\mathcal{J}^2 \ldots M_o^n\mathcal{J}^n = M_o^1M_o^2 \ldots M_o^n\mathcal{J}^1\mathcal{J}^2 \ldots \mathcal{J}^n \quad (43)$$

since the matrices $\mathcal{J}$ commute with all other matrices. The product $\mathcal{J}^1\mathcal{J}^2 \ldots \mathcal{J}^n$ is again a scalar matrix with the associated scalar $K^1K^2 \ldots K^n$. Here $K^1, K^2 \ldots K^n$ are the nonreciprocities of the two-ports $M^1, M^2 \ldots M^n$ respectively and the product $K^1K^2 \ldots K^n$ is the nonreciprocity of the overall two-port $M^1M^2 \ldots M^n$ so that one has the theorem:

The product of the nonreciprocities of tandem two-ports, equals the nonreciprocity of the "overall two-port".

It has been shown[4] that the nonreciprocity K(which is simply a complex number) is related to the parameters of various matrix two-port representations as follows

$$K = \sqrt{AD - BC} = \sqrt{S_{12}/S_{21}} = \sqrt{Z_{12}/Z_{21}} = \sqrt{Y_{12}/Y_{21}} = \sqrt{\tau_{11}\tau_{22} - \tau_{12}\tau_{21}} \quad . \quad (44)$$

As is usual, S stands for scattering, Z for impedance, Y for admittance and $\tau$ for scattering transfer. The theorem covering the product of nonreciprocities is consequently applicable to all these representations.

## Appendix II: Probe Excited Traveling Waves

In considering a reversed bridge, it is necessary to have a clear picture of the circumstances under which $a_1'$ and $a_2'$, the two traveling waves set up by a voltage probe in a waveguide, can be considered to be equal. When the probe is either perfectly straight, or is bent in such a manner that it lies completely in the transverse plane, there is no problem: The two waves are equal. When the probe is bent partially or completely in the direction of propagation the equality can no longer be assumed.

Consider the following Gedanken experiment with reference to Fig. 9. Let both source and detector be matched and let the bridge

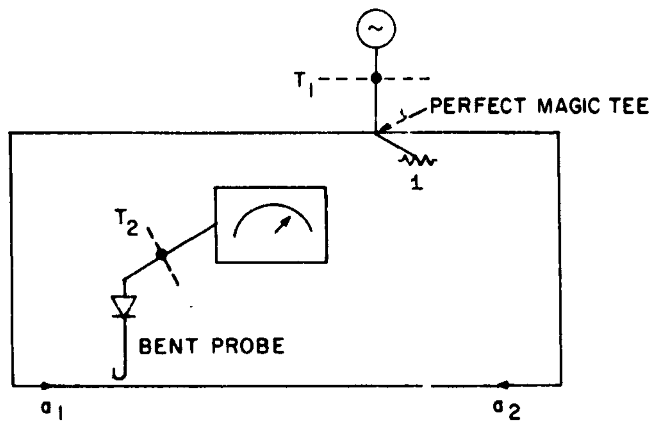


Fig. 9 Symmetric bridge with bent probe

be reciprocal and perfectly symmetrical about the magic tee except for the shape and location of the probe. The waves incident from the magic tee, $a_1$ and $a_2$, are consequently equal. Now move the probe to a voltage minimum. If the probe were straight, this would be a voltage zero to within the noise of the system. If the probe is bent, there still may exist an effective minimum location at which induced voltage is so small that it lies below the noise, or so slightly above the noise that it can be neglected. Under these circumstances the detected voltage is, for all practical purposes, zero. This situation is relatively common in the usual slotted line measurements. Now let source and detector be interchanged without moving the probe. The two-port between planes $T_1$ and $T_2$ is reciprocal so that the detected voltage may again be practically zero. This is possible only if the two waves incident on the magic tee are for practical purposes equal, which can be the case only if $a_1'$ and $a_2'$, the waves set up by the probe, have been very nearly equal. It is seen then that the two traveling waves set up by a voltage probe can be considered to be equal as long as the same probe is capable of measuring a voltage zero to within desired accuracy and the noise of the system.

## Appendix III: Summary of Measurement Procedure for Reversed Bridge

1. Short circuit planes $T_1$ and $T_2$ and measure $D_R$, $S_R$ and $\lambda_g$ (see Fig. 7).
2. Measure the length of a suitable waveguide, $\ell_o$ and insert it between planes $T_1$ and $T_2$. Balance the bridge.[5]

3. Set the variable phase shifter to some arbitrary setting $\phi_o$ and measure $D_o$ and $S_o$.

4. The values found above should satisfy the relation

$$D_o + S_o + \ell_o - D_R - S_R = n\lambda_g/2, \quad n = 0, \pm 1, \pm 2, \ldots$$

5. Replace the waveguide $\ell_o$ by the two-port to be measured and for each of a series of settings of $\phi$ measure D and $r_1$ with the left slotted line and S and $r_2$ with the other. Include the setting $\phi_o$ in the series. (Take $r_1$ and $r_2$, the VSWR, as positive only).

6. Compute

$$\overline{\Gamma}_1 = \frac{r_1 - 1}{r_1 + 1} e^{j2\kappa D} \quad \text{and} \quad \overline{\Gamma}_2 = \frac{r_2 - 1}{r_2 + 1} e^{j2\kappa S}$$

corresponding to each $\phi$ setting. The values corresponding to $\phi_o$ are designated $\overline{\Gamma}_{1o}$ and $\overline{\Gamma}_{2o}$. Values of $|\overline{\Gamma}_1|$ and $|\overline{\Gamma}_2|$ found in this way may either be the proper ones or the inverse of the proper values. Means for distinguishing between $|\Gamma|$ and its inverse are available elsewhere.[5]

7. Plot the points $\overline{\Gamma}_1$ and $\overline{\Gamma}_2$ on two sheets of polar coordinate paper and draw a circle through each.

8. Identify $|S_{11}|$, $\overline{\phi}_{11}$, $\psi_{1o}$ and $|S_{21}|$ (Note: $|C_1| = |C_2|$) on the $\overline{\Gamma}_1$ locus, and $|S_{22}|$, $\overline{\phi}_{22}$, $\psi_{2o}$ and $|S_{12}|$ on the $\overline{\Gamma}_2$ locus. See Fig. 8 for details.

9. Compute

$$\phi_{11} = \overline{\phi}_{11} - 2\kappa D_R \pm \pi$$

$$\phi_{22} = \overline{\phi}_{22} - 2\kappa S_R \pm \pi$$

$$\phi_{21} = \psi_{1o} - 2\kappa D_o - \kappa \ell_o$$

$$\phi_{12} = \psi_{2o} - 2\kappa S_o - \kappa \ell_o$$

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